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Ableitung an einer Stelle x_0

Aufgabe 1:

a) $f(x) = 2x^2 \qquad x_0 = 4$

$$\lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4} 2 \cdot (x+4) = \underline{16} \qquad \Rightarrow \qquad f'(4) = \underline{\underline{16}}$$

NR: $$\frac{2x^2 - 2 \cdot 16}{x-4} = \frac{2\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} = 2 \cdot (x+4)$$

b) $f(x) = \frac{6}{x} \qquad x_0 = -2$

$$\lim_{x \to -2} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2} \left(\frac{3}{x}\right) = \frac{3}{-2} = \underline{\underline{-\frac{3}{2}}} \qquad \Rightarrow \qquad f'(-2) = \underline{\underline{-\frac{3}{2}}}$$

NR: $$\frac{\frac{6}{x} + \frac{6}{2}}{x+2} = \frac{x\left(\frac{6}{x} + \frac{6}{2}\right)}{x\,(x+2)} = \frac{6+3x}{x\,(x+2)} = \frac{3\cancel{(2+x)}}{x\cancel{(x+2)}}$$

c) $f(x) = x^2 + 6x \qquad x_0 = 2$

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} (x+8) = \underline{\underline{10}} \qquad \Rightarrow \qquad f'(2) = \underline{\underline{10}}$$

NR: $$\frac{x^2 + 6x - (4 + 12)}{x-2} = \frac{x^2 - 4}{x-2} + \frac{6x - 12}{x-2}$$

$$= \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} + \frac{6 \cdot \cancel{(x-2)}}{\cancel{(x-2)}} = x + 2 + 6 = x+8$$

d) $f(x) = \sqrt{x} \qquad x_0 = 3$

$$\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{1}{\sqrt{x} + \sqrt{3}} = \underline{\underline{\frac{1}{2 \cdot \sqrt{3}}}} \qquad \Rightarrow \qquad f'(3) = \underline{\underline{\frac{1}{2 \cdot \sqrt{3}}}}$$

NR: $$\frac{\sqrt{x} - \sqrt{3}}{x-3} = \frac{\cancel{(\sqrt{x} - \sqrt{3})}}{\cancel{(\sqrt{x} - \sqrt{3})}(\sqrt{x} + \sqrt{3})} = \frac{1}{\sqrt{x} + \sqrt{3}}$$

Aufgabe 2:

a) $f(x) = \frac{1}{2} x^2; \quad x_0 = 1$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{1}{2}(x+1) = \underline{\underline{1}} \qquad \Rightarrow \qquad f'(1) = 1$$

NR: $$\frac{\frac{1}{2}x^2 - \frac{1}{2} \cdot 1}{x-1} = \frac{\frac{1}{2}(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \frac{1}{2}(x+1)$$

b) $f(x) = x^2 - 2x; \quad x_0 = 1$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} (x-1) = 0 \qquad \Rightarrow \qquad \underline{\underline{f'(1) = 0}}$$

NR: $$\frac{x^2 - 2x - (1-2)}{x-1} = \frac{x^2 - 1}{x-1} - \frac{2x-2}{x-1}$$

$$= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} - \frac{2\cancel{(x-1)}}{\cancel{(x-1)}} = x + 1 - 2$$

c) $f(x) = \frac{1}{x+1}; \quad x_0 = 1$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} -\frac{1}{2(x+1)} = -\frac{1}{4} \qquad \Rightarrow \qquad \underline{\underline{f'(1) = -\frac{1}{4}}}$$

NR: $$\frac{\frac{1}{x+1} - \frac{1}{2}}{x-1} = \frac{2 \cdot (x+1)\left(\frac{1}{x+2} - \frac{1}{2}\right)}{2(x+1)(x-1)} = \frac{2 - (x+1)}{2(x+1)(x-1)}$$

$$= \frac{-\cancel{(-1+x)}}{2(x+1)\cancel{(x-1)}} = -\frac{1}{2(x+1)}$$

d) $f(x) = \frac{x}{x+1}; \quad x_0 = 1$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{1}{2(x+1)} = \frac{1}{4} \qquad \Rightarrow \qquad f'(1) = \underline{\underline{\frac{1}{4}}}$$

NR: $$\frac{\frac{x}{x+1} - \frac{1}{2}}{x-1} = \frac{2 \cdot (x+1)\left(\frac{x}{x+1} - \frac{1}{2}\right)}{2 \cdot (x+1)(x-1)} = \frac{2x - (x+1)}{2(x+1)(x-1)} = \frac{\cancel{(x-1)}}{2(x+1)\cancel{(x-1)}}$$

Aufgabe 3:

a) $f(x) = 2x^2$; $x_0 = 4$

$$f'(4) = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h} = \lim_{h\to 0} (16+2h) = \underline{\underline{16}} \quad \Rightarrow \quad f'(4) = \underline{\underline{16}}$$

NR: $$\frac{2\cdot(4+h)^2 - 2\cdot 4^2}{h} = \frac{2\cdot 4^2 + 2\cdot 8h + 2h^2 - 2\cdot 4^2}{h} = \frac{\cancel{h}(16+2h)}{\cancel{h}}$$

b) $f(x) = \frac{6}{x}$; $x_0 = -2$

$$f'(-2) = \lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0} \frac{3}{h-2} = -\frac{3}{2} \quad \Rightarrow \quad f'(-2) = \underline{\underline{-\frac{3}{2}}}$$

NR: $$\frac{\frac{6}{-2+h} - \frac{6}{-2}}{h} = \frac{\left(\frac{6}{h-2} + 3\right)\cdot h-2}{h\cdot(h-2)} = \frac{6+3h-6}{h(h-2)} = \frac{3\cancel{h}^1}{\cancel{h}(h-2)}$$

c) $f(x) = x^2 + 6x$; $x_0 = 2$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} (10+h) = 10 \quad \Rightarrow \quad f'(2) = 10$$

NR: $$\frac{(2+h)^2 + 6(2+h) - (4+12)}{h} = \frac{\cancel{4}+4h+h^2+\cancel{12}+6h-\cancel{4}-\cancel{12}}{h}$$
$$= \frac{\cancel{h}^1(10+h)}{\cancel{h}_1} = 10+h$$

d) $f(x) = \sqrt{x}$ $x_0 = 3$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{1}{\sqrt{3+h}+\sqrt{3}} = \underline{\underline{\frac{1}{2\cdot\sqrt{3}}}}$$

NR: $$\frac{\sqrt{3+h}-\sqrt{3}}{h} = \frac{(\sqrt{3+h}+\sqrt{3})(\sqrt{3+h}-\sqrt{3})}{(\sqrt{3+h}+\sqrt{3})\cdot h} = \frac{3+h-3}{(\sqrt{3+h}+\sqrt{3})\cdot h} = \frac{1}{\sqrt{3+h}+\sqrt{3}}$$

Aufgabe 4:

a) $f(x) = \frac{1}{2}x^2$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} (x_0+2h) = \underline{\underline{x_0}} \qquad f'(x_0) = x_0$$

NR: $$\frac{\frac{1}{2}(x_0+h) - \frac{1}{2}x_0^2}{h} = \frac{\frac{1}{2}(\cancel{x_0^2}+2hx_0+h^2-\cancel{x_0^2})}{h}$$
$$= \frac{\frac{1}{2}h(2x_0+h)}{h} = x_0 + \frac{1}{2}h$$

b) $f(x) = \sqrt{x}$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \left(\frac{1}{\sqrt{x_0+h}+\sqrt{x_0}}\right) = \underline{\underline{\frac{1}{2\cdot\sqrt{x_0}}}} \quad \Rightarrow \quad f'(x_0) = \frac{1}{2\sqrt{x_0}}$$

NR: $$\frac{\sqrt{x_0+h}-\sqrt{x_0}}{h} = \frac{(\sqrt{x_0+h}+\sqrt{x_0})(\sqrt{x_0-h}-\sqrt{x_0})}{(\sqrt{x_0+h}+\sqrt{x_0})\,h} = \frac{x_0+h-x_0}{(\sqrt{x_0+h}+\sqrt{x_0})\cdot h}$$
$$= \frac{1}{\sqrt{x_0+h}+\sqrt{x_0}}$$

c) $f(x) = \frac{1}{x+1}$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{-1}{(x_0+h+1)(x_0+1)} = \underline{\underline{-\frac{1}{(x_0+1)^2}}}$$

NR: $$\frac{\frac{1}{x_0+h+1} - \frac{1}{x_0+1}}{h} = \frac{(x_0+h+1)(x_0+1)\left(\frac{1}{x_0+h+1} - \frac{1}{x_0+1}\right)}{(x_0+h+1)(x_0+1)\cdot h}$$
$$= \frac{x_0+1-(x_0+h+1)}{(x_0+h+1)(x_0+1)\cdot h} = \frac{-\cancel{h}^1}{(x_0+h+1)(x_0+1)\cdot\cancel{h}_1}$$

d) $f(x) = \frac{x}{x+1}$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{1}{(x_0+h+1)(x_0+h)} = \frac{1}{(x_0+1)^2}$$

NR: $$\frac{\frac{x_0+h}{x_0+h+1} - \frac{x_0}{x_0+1}}{h} = \frac{(x_0+1)(x_0+h) - (x_0+h+1)(x_0)}{(x_0+h+1)(x_0+1)\cdot h}$$
$$= \frac{\cancel{x_0^2}+h+\cancel{hx_0}+\cancel{x_0}-\cancel{x_0^2}-\cancel{hx_0}-\cancel{x_0}}{(x_0+h+1)(x_0+1)\cdot\cancel{h}_1} = \frac{1}{(x_0+h+1)(x_0+1)}$$

Aufgabe 5:

a) $f(x)=\frac{1}{4}x^2 \qquad x_0=2$

$f(2)=1 \qquad P(2|1)$

NR: $\frac{\frac{1}{4}(2+h)^2-1}{h} = \frac{1+h+\frac{1}{4}h^2-1}{h}$

$= 1+\frac{1}{4}h$

Tangente: $t: y=mx+c$

$m=f'(2)=\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}=1$

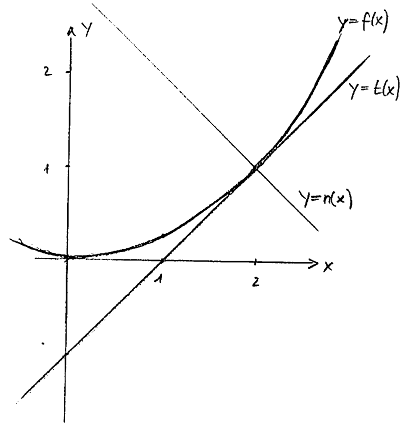
$t(2)=1 \Rightarrow 1=1\cdot 2+c \quad |-2$

$c=-1$

$\Rightarrow \quad t(x)=x-1$

Normale: $m_n=-\frac{1}{m_t}=-1$

$n(x)=-x$



b) $f(x)=\sqrt{2x} \qquad x_0=2$

$f(2)=2 \qquad P(2|2)$

NR: $\frac{\sqrt{2\cdot(2+h)}-\sqrt{4}}{h} = \frac{4+2h-4}{h\left(\sqrt{4+2h}+\sqrt{4}\right)}$

$= \frac{2}{\sqrt{4+2h}+2}$

$m=f'(2)=\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}=\frac{2}{4}=\underline{\underline{\frac{1}{2}}}$

$t(2)=2 \Rightarrow 2=\frac{1}{2}\cdot 2+c \quad |-1$

$c=1$

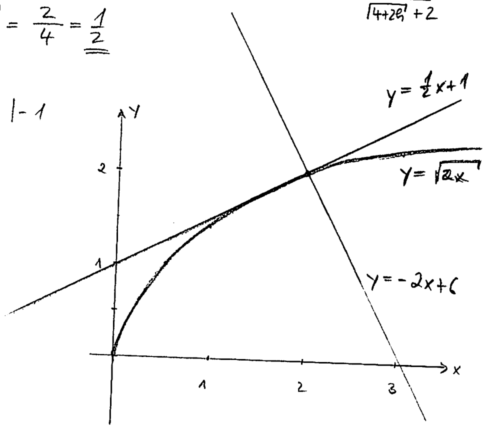
$\Rightarrow \quad \underline{\underline{t(x)=\frac{1}{2}x+1}}$

$n(x)=-2x+c$

$n(2)=2 \Rightarrow 2=-4+c$

$c=6$

$n(x)=-2x+6$



c) $f(x)=\frac{1}{x+2} \qquad x_0=1{,}5$

$f(1{,}5)=\frac{1}{1{,}5+2}=\frac{1}{3{,}5}=\frac{2}{7}$

$m=f'(1{,}5)=\lim\limits_{h\to 0}\frac{\frac{1}{1{,}5+h+2}-\frac{2}{7}}{h}$

$=-\frac{2}{7\cdot 3{,}5}=-\underline{\underline{\frac{4}{49}}}\approx -0{,}0816$

NR: $\frac{1}{h}\cdot\left(\frac{1}{3{,}5+h}-\frac{2}{7}\right)$

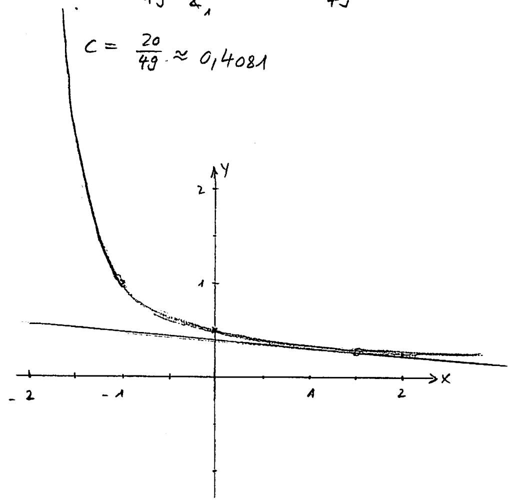
$=\frac{1}{h}\cdot\left(\frac{7}{7(3{,}5+h)}-\frac{2\cdot(3{,}5+h)}{7(3{,}5+h)}\right)$

$=\frac{1}{h}\cdot\frac{-2h}{7\cdot(3{,}5+h)}=-\frac{2}{7(3{,}5+h)}$

$t(1{,}5)=\frac{2}{7} \Rightarrow \frac{2}{7}=-\frac{4}{49}\cdot 1{,}5+c$

$\frac{2}{7}=-\frac{\cancel{4}^{2}}{49}\cdot\frac{3}{\cancel{2}_{1}}+c \quad |+\frac{6}{49}$

$c=\frac{20}{49}\approx 0{,}4081$



$n(x)=+\frac{49}{4}x+c$

$n(1{,}5)=\frac{2}{7} \Rightarrow \frac{2}{7}=\frac{49}{4}\cdot\frac{3}{2}+c$

$\Rightarrow c=-\frac{1013}{56}$

$\Rightarrow n(x)=\frac{49}{4}x-\frac{1013}{56}$

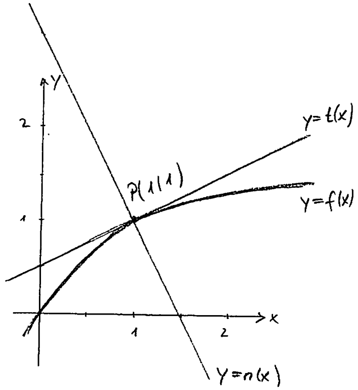
d) $f(x) = \frac{2x}{x+1}$ $\quad x_0 = 1$

$f(1) = \frac{2\cdot 1}{1+1} = \frac{2}{2} = 1$

$m = f'(1) = \lim\limits_{h \to 0} \frac{\frac{2(1+h)}{1+h+1} - 1}{h} = \underline{\underline{\frac{1}{2}}}$

<u>NR:</u> $\frac{1}{h} \cdot \left(\frac{2+2h}{2+h} - \frac{2+h}{2+h} \right)$

$= \frac{1}{h} \cdot \left(\frac{h}{2+h} \right) = \frac{1}{2+h}$



$t(1) = 1 \Rightarrow \quad 1 = \frac{1}{2} \cdot 1 + c \quad | -\frac{1}{2}$

$c = \frac{1}{2}$

$t(x) = \frac{1}{2} \cdot x + \frac{1}{2}$

$n(1) = 1 \quad \Rightarrow \quad 1 = -2 \cdot 1 + c \quad | +2$

$c = 3$